NBU-003-1262001 Seat No. _______

M. Phil. (Sem. II) (CBCS) Examination

April / May - 2017

Mathematics : 20001

(Topology) (New Course)

Faculty Code : 003

Subject Code : 1262001

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions:

(1) There are five questions in this paper.

(2) Each question carries 14 marks.

(3) All questions are compulsory

1 Fill in the blanks: (Each question carries two marks)

(a) If f: X → ℝ is a continuous function then the zero set of f is a countable intersection of ………… sets.

(b) Every space X is ………. embedded in its Stone – Cech compactification.

(c) If I is a Z – ideal which contains a prime ideal then I is a ………..

(d) In ℝ with lower limit topology every closed subset is ………..embedded in ℝ .

(e) In C (ℕ) every ideal is a ………..ideal.

(f) If A is a commutative Banach Algebra with multiplicative identity then its maximal ideal space is ………….. and …………….

(g) Then function J (n) is contained in every……….. ideal in C^* (ℕ).

2 Attempt any two of the following:

(a) State and prove the necessary and sufficient condition under which a C^* - embedded subset of a space is C – embedded. 7

(b) Prove that an ideal M is a maximal ideal if and only if Z(M) is a Z – ultra filter. 7

(c) (i) Give an example of an ideal in C^* (ℝ) which is not a fixed ideal. 7

(ii) Prove for any ideal I, $Z^{-1}(Z(I))$ is a Z – ideal.

3 All are compulsory:

(a) Give an example of a Z – ideal which is contained in a unique maximal ideal in C ($\mathbb{R}$) 6

(b) Prove that intersection of two maximal ideals in C (X) cannot be a prime ideal. 4

(c) Give an example of an ideal in $C^*(\mathbb{N})$ which is not the intersection of any ideal of C ($\mathbb{N}$) with $C^*(\mathbb{N})$. 4

OR

3 All are compulsory:

(a) Prove that the countable intersection of zero sets is a zero set. 5

(b) Give an example of a subset of $\mathbb{R}$ which is not C^* - embedded in $\mathbb{R}$. 4

(c) Prove that intersection of any family of Z – ideal is a Z – ideal. 5

4 Attempt any two of the following:

(a) Prove that a space X is compact if and only if every maximal ideal in C^* (X) is fixed. 7

(b) Suppose X is a dense subspace of T and X is C^* - embedded in T. Prove that 7

(i) If Z_1 and Z_2 are disjoint zero sets in X then $Cl_T(Z_1)$ and $Cl_T(Z_2)$ are disjoint.

(ii) If Z_1 and Z_2 are zero sets in X then $Cl_T(Z_1 \cap Z_2) = Cl_T Z_1 \cap Cl_T Z_2$.

(c) Let X be a compact Hausdorff space and C (X) denote the Banach Algebra of all complex valued continuous functions on X. Prove that 7

(i) For any closed subset F of X the set $I_F = \{f \in C(X) / f(F) = \{0\}\}$ is a closed ideal in C (X).

(ii) Prove that the closure of any ideal in C (X) is an ideal in C (X).

5 Do as directed: (Each question carries two marks)

(a) Give reason: why $\mathbb{R} \setminus \{0\}$ is not a zero set ?

(b) Give a continuous function f: $\mathbb{R} \rightarrow \mathbb{R}$ for which Z (f) is a countable infinite set.

(c) Suppose $f \in C(X)$ and $A = \{x \in X / f(x) \geq \sqrt{2}\}$. Is A a zero set ? Give reason.

(d) Suppose f(x) = x for all x in $\mathbb{R}$.. Let I be the principal ideal generated by f(x). Give a function g in C (X) such that $g \in Z^{-1}(Z(I))$ but $g \notin I$.

(e) Give an example of a maximal ideal in C ($\mathbb{R}$) which is fixed.

(f) Give an example of a C^* - embedded subset of $\beta \mathbb{N}$, where $\beta \mathbb{N}$ denotes the stone – Cech compactification of $\mathbb{N}$.

(g) Give a continuous function f: $\mathbb{N} \rightarrow \mathbb{R}$ such that f cannot be extended to a continuous function g: $\beta \mathbb{N} \rightarrow \mathbb{R}$.